UNIT-2 ELECTROMAGNETIC FIELD THEORY

Basics of EM Theory :-

(i) Electric field :- When two charge particles are put together then fore exerts by one charge particle to other and vice-versa within a region is called electric field.

(ii) Magnetic field :- A region in which a charged body feels magnetization is called magnetic field.

(iii) Permittivity :- Strength of the material to store electric charge. Its SI unit is farad/meter. Its value 8.85×10^{-12} for free space or vaccum (ϵ_0).

(iv) Permeability :- Ability of the material to store magnetization. Its SI unit Henary/meter and its value $4\pi \times 10^{-7}$ H/m for free space (μ_0).

(v) Gauss' law of electrostatics :- The total electric flux (ϕ_E) enclosed over a surface is $1/\epsilon$ times of the charge enclosed other that surface.

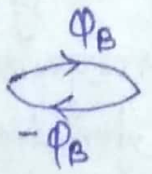
$$\boxed{\phi_E = \frac{q}{\epsilon} = \oint_S E.ds}$$

(vi) Gauss' law of magnetostatics :- The total magnetic flux (ϕ_B) enclosed over a surface is always be equal to zero.

$$\boxed{\phi_B = \oint B.ds = 0}$$

or

flux entering and leaving from a surface is same.



Total flux $= \phi_B + (-\phi_B) = \phi_B - \phi_B = 0$

Ampere's Circuital Law :- Line Integral of magnetic field (B) is equal to μ times of the current in the circuit.

$$\boxed{\oint_L B \cdot dl = \mu I}$$

Faraday's law of induction :- An emf(e) induced in a circuit is equal to the negative rate change of flux in the circuit.

$$\boxed{e = -\frac{d\phi}{dt}} \quad \boxed{e = -\frac{d\phi_E}{dt}} \text{ and } \boxed{e = -\frac{d\phi_B}{dt}}$$

{in case of electric flux} {In case of magnetic flux}

Del operator (∇) :- It is an operator which is not a vector itself but when operates over a scaler function results a vector.

$$\boxed{\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}}$$

∴ Let $f(x,y,z)$ is a scaler function then $\nabla f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$

Gradient :- It defines the change in the function when the variables of function are changed.

Let $f(x,y,z)$ is a function then its gradient

$$\boxed{\nabla f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}}$$ The gradient is a vector.

Divergence :- It defines how the function diverge from a point. The divergence is a scaler quantity.

diverge Converge

$$\boxed{\nabla \cdot f = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}}$$

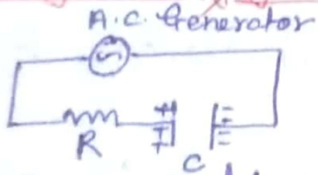
If divergence is -ve it means function is converging.

Curl :- It defines how function is rotated at a point. It is a vector quantity.

$$\nabla \times f = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix} = \hat{i}\left(\frac{\partial f_z}{\partial x} - \frac{\partial f_y}{\partial z}\right) - \hat{j}\left(\frac{\partial f_z}{\partial x} - \frac{\partial f_x}{\partial z}\right) + \hat{k}\left(\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y}\right)$$

Displacement Current (I_d) :- The flow of current produces magnetic field. The change in magnetic field is also produces magnetic field in the capacitor. It generates a current same as conduction current. This current is called displacement current.

Physical Significance :- Consider a circuit which consists of an alternating source alongwith the a resistance (R) and capacitor (C) in series.



Current flows in the circuit is called conduction current because its generates by the movement of electrons. This current produces a magnetic field. If $I_c = 0$ then $B = 0$. Now the question is that there is a dielectric inside the capacitor i.e. no current inside the capacitor but still there is a magnetic field. Maxwell suggested that accumulation of charge on the plates of capacitor produces a potential difference which creates change in electric field as result a current is buildup inside the capacitor. This is known as displacement current (I_d). It exists only inside the capacitor until electric field is changed.

The conduction current $(I_c) = \frac{V}{R}$ — (1) Displacement Current (I_d) $= \frac{dq}{dt} = \frac{d(CV)}{dt}$ $\because q = CV$

$I_d = \frac{\epsilon_0 A}{d} \frac{d(Ed)}{dt}$ $\because C = \frac{\epsilon_0 A}{d}, V = \frac{E}{d}$

$I_d = \epsilon_0 A \frac{dE}{dt} = \epsilon_0 A \frac{d}{dt}\left(\frac{D}{\epsilon_0}\right)$ Where d is separation between the plates of the capacitor.

$I_d = A \frac{dD}{dt}$ $\boxed{J_d = \frac{I_d}{A} = \frac{dD}{dt}}$

$J_d \rightarrow$ Current density due to displacement. $D \rightarrow$ Electric displacement vector

Now total current becomes $(I) = I_c + I_d$
So total current density $(J) = J_c + J_d$
Hence the Ampere's law will be modified as

$$\boxed{\oint_L B \cdot dl = \mu (I_c + I_d)}$$

This is called modified Ampere' Circuital law or Ampere-Maxwell's law. It is valid for steady state as well as alternating state.

Q:- Show that how Ampere's law modified by using the concept of displacement current.

Maxwell's Equations :- Maxwell relates the electric & magnetic field and gave four equations.

(i) Maxwell's Ist Equation in differential form for a Conducting medium :-

According to Gauss law of electrostatics,

$$\oint_S E \cdot ds = \phi_E = \frac{q}{\epsilon} \quad \text{---} \ (1)$$

Let the charge is not only distributed over a surface but also over the volume V.

$$q = \oint_V \rho \, dv \quad \text{where } \rho \text{ - volume charge density} \quad \text{---} \ (2)$$

$$\boxed{\oint_S E \cdot ds = \frac{1}{\epsilon} \oint_V \rho \, dv} \rightarrow$$ from (1) & (2) Integral form of Maxwell's Ist Eqn

According to Gauss divergence Theorem

$$\oint_S E \cdot ds = \int_V (\nabla \cdot E) \, dv \quad \text{---} \ (3)$$

from (2) & (3)

$$\int_V (\nabla \cdot E) \, dv = \frac{1}{\epsilon} \int_V \rho \, dv$$

$$\boxed{\nabla\cdot E = \frac{\rho}{\epsilon}} \quad \text{or} \quad \nabla\cdot\frac{D}{\epsilon} = \frac{\rho}{\epsilon} \Rightarrow \boxed{\nabla\cdot D = \rho}$$

Maxwell 2nd Equation :- According to Gauss' law of magneto-statics,

$$\oint_S B\cdot dS = 0 \quad \text{—— (1)}$$

According to Gauss' divergence Theorem

$$\boxed{\oint_S B\cdot dS = \int_V (\nabla\cdot B)\, dv} \quad \text{—— (2)}$$

Integral form of Maxwell's 2nd Equation

from (1) + (2)

$$\boxed{\int_V (\nabla\cdot B)\, dv = 0}$$

$$\boxed{\nabla\cdot B = 0} \quad \text{or} \quad \nabla\cdot(\mu H) = 0 \Rightarrow \boxed{\nabla\cdot H = 0} \quad \because B = \mu H$$

Maxwell's 3rd Equation :- According to Faraday's law of induction,

$$e = -\frac{d\phi_B}{dt} \quad \text{—— (1)}$$

The work done in the circuit is also equal to the emf induce in the circuit.

$$e = \oint_L E\cdot dL \quad \text{—— (2)}$$

from (1) & (2)

$$\oint_L E\cdot dL = -\frac{d}{dt}\phi_B$$

$$\boxed{\oint_L E\cdot dL = -\frac{d}{dt}\oint_S B\cdot dS} \quad \because \phi_B = \oint_S B\cdot dS \quad \text{—— (3)}$$

Integral form of Maxwell's 3rd Equation

According to Stoke's Theorem

$$\oint_L E \cdot dL = \oint_S (\nabla \times E) \cdot ds \quad \text{--- (4)}$$

from (3) & (4)

$$\oint_S (\nabla \times E) \cdot ds = -\frac{d}{dt}\oint_S B \cdot ds$$

$$\boxed{\nabla \times E = -\frac{dB}{dt}}$$

(IV) Maxwell's 4th Equation :-

According to modified Ampere's law

$$\oint_L B \cdot dL = \mu(I_c + I_d) \quad \text{--- (1)}$$

$$I_c = \oint_S J_c \cdot ds \quad \& \quad I_d = \oint_S J_d \cdot ds$$

$$\oint_L B \cdot dL = \mu\left(\oint_S J_c \cdot ds + \oint_S J_d \cdot ds\right)$$

$$\oint_L B \cdot dL = \mu \oint_S (J_c + J_d) \cdot ds$$

$$\boxed{\oint_L B\,dL = \mu \oint_S \left(J + \frac{\partial D}{\partial t}\right) \cdot ds} \quad \text{--- (2)} \quad \because J_c = J, \; J_d = \frac{\partial D}{\partial t}$$

Integral form of 4th Maxwell's Equation

Acc. to Stoke's Theorem

$$\oint_L B \cdot dL = \oint_S (\nabla \times B) \cdot ds \quad \text{--- (3)}$$

from (2) & (3)

$$\oint_S (\nabla \times B) \cdot ds = \mu \oint_S \left(J + \frac{\partial D}{\partial t}\right) \cdot ds \Rightarrow \boxed{\nabla \times B = \mu\left(J + \frac{\partial D}{\partial t}\right)}$$

or

$$\nabla \times (\mu H) = \mu\left(J + \frac{\partial D}{\partial t}\right)$$

$$\boxed{\nabla \times H = J + \frac{\partial D}{\partial t}}$$

Physical Significance of Maxwell's Equations

(i) $\nabla \cdot E = \rho/\epsilon$

This equation represents Gauss' law of electrostatics and time independent.

(ii) $\nabla \cdot B = 0$

This equation represents Gauss' law of magnetostatics and also time independent.

(iii) $\nabla \times E = -\frac{\partial B}{\partial t}$

This equation represents Faraday's law of Induction & time dependent.

(iv) $\nabla \times B = \mu\left(J + \frac{\partial D}{\partial t}\right)$

This equation represents Modified Ampere's circuital law in which the term $\frac{\partial D}{\partial t}$ represents the concept of displacement current. This equation is also time dependent.

Maxwell's Equation for free space or vacuum :—

We know that for free space $\sigma = \rho = J = 0$

So equations becomes

(i) $\nabla \cdot E = \rho/\epsilon = 0 \Rightarrow \boxed{\nabla E = 0}$

(ii) $\boxed{\nabla \cdot B = 0}$

(iii) $\boxed{\nabla \times E = -\frac{\partial B}{\partial t}}$

(iv) $\nabla \times B = \mu\left(J + \frac{\partial D}{\partial t}\right) = \mu\left(0 + \frac{\partial D}{\partial t}\right)$

$$\boxed{\nabla \times B = \mu \frac{\partial D}{\partial t}}$$

Note :— Remember

Gauss' Divergence Theorem

Surface integral of any vector is equal to the volume integral of the divergence that vector

$$\boxed{\oint E \cdot ds = \int_V (\nabla \cdot E)\, dv}$$

Stoke's Theorem :- Line integral of any vector is equal to the surface integral of the curl of that vector

$$\boxed{\oint_L E \cdot dL = \oint_S (\nabla \times E) \cdot ds}$$

Continuity Equation for electromagnetics :- It defines the charge in and out from a volume V.

Let I is current

Then $I = -\frac{dq}{dt}$ (due to electrons) — (1)

We have $I = \oint_S J \cdot ds$ & $q = \int_V \rho \, dv$ where J - Current density, $\rho \to$ volume charge density

Now, $\oint_S J \cdot ds = -\frac{d}{dt}\int_V \rho \, dv$ — (2)

Acc. to Gauss' divergence Theorem

$\oint_S J \cdot ds = \int_V (\nabla \cdot J) \cdot dv$ — (3)

from (2) & (3)

$\int_V (\nabla \cdot J) \, dv = -\frac{d}{dt}\int_V \rho \, dv \Rightarrow \nabla \cdot J = -\frac{\partial \rho}{\partial t}$ or $\boxed{\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0}$

2nd Method by using Maxwell's Equation :-

We know that

$\nabla \times H = J + \frac{\partial D}{\partial t}$ — (1) Taking the divergence of Equation (1)

$\nabla \cdot (\nabla \times H) = \nabla \cdot \left(J + \frac{\partial D}{\partial t}\right)$

We know that divergence of curl of any vector is always be zero.

$\nabla \cdot J + \frac{\partial (\nabla \cdot D)}{\partial t} = 0 \Rightarrow \boxed{\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0}$ $\{ \because \nabla \cdot D = \rho$

Physical Significance :- This equation defines divergence of current density is equal to the -ve rate change of volume charge density. It gives information of transport of any quantity from one place to other place.

Poynting Vector :- Energy radiated per unit area & per unit time ~~from the~~ in the direction perpendicular to the propagation of electromagnetic wave is called Poynting vector & denoted by P or S.

Poynting Theorem :- A mathematical expression which relates the electric & magnetic field is called Poynting theorem.

$$\boxed{\vec{S} = \vec{E} \times \vec{B}} \text{ or } \boxed{\vec{S} = \frac{1}{\mu}(\vec{E} \times \vec{H})}$$

Derivation of energy in electromagnetic field or Poynting Theorem:

We have $\nabla \times H = J + \epsilon \frac{\partial E}{\partial t}$ —— ①

Taking dot product of E on both sides

$$E\cdot(\nabla \times H) = E\cdot\left(J + \epsilon \frac{\partial E}{\partial t}\right)$$

$$\nabla\cdot(E \times H) = H\cdot(\nabla \times E) - E\cdot(\nabla \times H)$$

$\because \nabla\cdot(A \times B) = B\cdot(\nabla \times A) - A\cdot(\nabla \times B)$

$A\cdot(\nabla \times B) = B\cdot(\nabla \times A) - \nabla\cdot(A \times B)$

$$H\cdot(\nabla \times E) - \nabla\cdot(E \times H) = E\cdot J + \epsilon \frac{\partial(E\cdot E)}{\partial t} \quad \text{—— ②}$$

$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t}$ from Maxwell's 3rd Equation

$$H\cdot(\nabla \times E) = -\mu \frac{\partial(H\cdot H)}{\partial t}$$

$$-\mu\left(\frac{\partial(H\cdot H)}{\partial t}\right) - \nabla\cdot(E \times H) = E\cdot J + \epsilon \frac{\partial(E\cdot E)}{\partial t} \quad \text{—— ③}$$

$$\frac{\partial(H\cdot H)}{\partial t} = H\cdot\frac{\partial H}{\partial t} + H\cdot\frac{\partial H}{\partial t} = 2H\cdot\frac{\partial H}{\partial t} = \frac{\partial H^2}{\partial t}$$

$H\cdot\frac{\partial H}{\partial t} = \frac{1}{2}\frac{\partial H^2}{\partial t}$ Similarly $E\cdot\frac{\partial E}{\partial t} = \frac{1}{2}\frac{\partial E^2}{\partial t}$ Put in ③

$$-\mu\left[\frac{1}{2}\frac{\partial H^2}{\partial t}\right] - \nabla\cdot(E \times H) = E\cdot J + \epsilon\left[\frac{1}{2}\frac{\partial E^2}{\partial t}\right]$$

$$-\nabla\cdot(E \times H) = E\cdot J + \epsilon\left[\frac{1}{2}\frac{\partial E^2}{\partial t}\right] + \mu\left[\frac{1}{2}\frac{\partial H^2}{\partial t}\right]$$

$$-\nabla\cdot(E \times H) = E\cdot J + \frac{\partial}{\partial t}\left[\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right]$$

$$\nabla\cdot(E \times H) = -E\cdot J - \frac{\partial}{\partial t}\left[\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right]$$

Integrate it over a volume V.

$$\int_V \nabla\cdot(E \times H)\,dV = -\int_V (E\cdot J)\,dV - \frac{\partial}{\partial t}\int_V \left[\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right] dV \quad \text{—— ④}$$

Using Gauss divergence Theorem

$$\oint_S (E \times H)\cdot ds = \int_V \nabla\cdot(E \times H)\,dV \quad \text{—— ⑤}$$

from ④ & ⑤

$$\boxed{\oint_S (E\times H)\cdot ds = -\int_V (E\cdot J)\,dV - \frac{\partial}{\partial t}\int_V \left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right) dV}$$

(i) The term $\int_V (E\cdot J)\,dV$ represents power dissipated at any instant in volume V.

(ii) $-\frac{\partial}{\partial t}$ represents rate at which stored energy is decreasing.

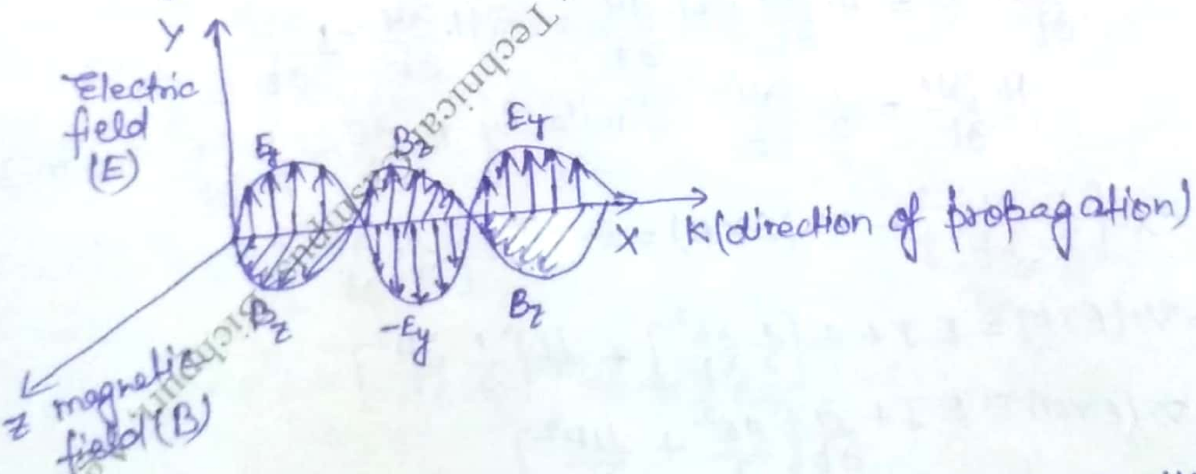
(iii) $\int_V \left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right) dV$ represents energy due to electric & magnetic field in volume V.

(iv) $\oint_S (E\times H)\cdot ds$ represents rate of flow of energy through the surface.

$E\times H = S$ measure of rate of flow of energy per unit area.

Watt/m^2.

Electromagnetic Wave :- An electromagnetic wave is a composition of oscillating electric and magnetic fields.



Electromagnetic wave = oscillating electric field + Oscillating magnetic field.

In an EM wave electric field, & magnetic field and direction of propagation are perpendicular to each other. It means EM waves are transverse in nature. These waves are travelled with the speed of light. The spectrum of EM radiation varies from gamma rays to radio waves.

Electromagnetic Waves in vacuum or free space :-

We have Maxwell's Equations

$$\nabla\cdot E = \rho/\epsilon \;,\quad \nabla\cdot B = 0 \;,\quad \nabla\times E = -\frac{\partial B}{\partial t} \;,\quad \nabla\times B = \mu\left(J + \frac{\partial D}{\partial t}\right)$$

for free space There is no charge so $\rho = 0$, $J = 0$ & $\epsilon = \epsilon_0$, $\mu = \mu_0$

Now Maxwell's Equations become

$$\nabla\cdot E = 0 \quad \text{or} \quad \nabla\cdot D = 0 \quad \text{——} \quad (1)$$

$$\nabla\cdot B = 0 \quad \text{——} \quad (2)$$

$$\nabla\times E = -\frac{\partial B}{\partial t} \quad \text{——} \quad (3)$$

$$\nabla\times B = \mu_0\frac{\partial D}{\partial t} \quad \text{——} \quad (4)$$

(a) EM wave equation in terms of E :

From (3)

Taking the curl of both sides

$$\nabla\times(\nabla\times E) = -\frac{\partial}{\partial t}(\nabla\times B) \quad \text{——} \quad (5)$$

We have the vector identity

$$\nabla\times(\nabla\times E) = \nabla(\nabla\cdot E) - \nabla^2 E$$

$$= 0 - \nabla^2 E \quad \because \nabla\cdot E = 0$$

$$\rightarrow -\nabla^2 E = -\frac{\partial}{\partial t}(\nabla\times B)$$

$$-\nabla^2 E = -\frac{\partial}{\partial t}\left[\mu_0\frac{\partial D}{\partial t}\right] \Rightarrow \nabla^2 E = \mu_0\frac{\partial^2}{\partial t^2}(\epsilon_0 E) \quad \because D = \epsilon_0 E$$

$$\boxed{\nabla^2 E = \mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2}}$$

(b) EM wave equation in terms of B :-

From (4)

$$\nabla\times B = \mu_0\frac{\partial D}{\partial t} = \mu_0\epsilon_0\frac{\partial E}{\partial t} \quad \text{——} \quad (6)$$

Taking the curl of both sides

$$\nabla\times(\nabla\times B) = \mu_0\epsilon_0\frac{\partial}{\partial t}(\nabla\times E)$$

$$\nabla\times(\nabla\cdot B) - \nabla^2 B = \mu_0\epsilon_0\frac{\partial}{\partial t}(\nabla\times E)$$ by using the vector identity

$$0 - \nabla^2 B = -\mu_0\epsilon_0\frac{\partial^2 B}{\partial t^2}$$ from (2) & (3)

$$\boxed{\nabla^2 B = \mu_0\epsilon_0\frac{\partial^2 B}{\partial t^2}}$$

EM wave equation in terms of H :–

from (4)

$$\nabla\times B = \mu_0\frac{\partial D}{\partial t} \Rightarrow \nabla\times\mu_0 H = \mu_0\frac{\partial \epsilon_0 E}{\partial t} \quad \because B=\mu_0 H,\ D=\epsilon_0 E$$

$$\nabla\times H = \epsilon_0\frac{\partial E}{\partial t}$$

Taking the curl of both sides

$$\nabla\times(\nabla\times H) = \epsilon_0\frac{\partial}{\partial t}(\nabla\times E)$$

$$\nabla\times(\nabla\cdot H) - \nabla^2 H = \epsilon_0\frac{\partial}{\partial t}(\nabla\times E)$$ by using vector identity

$$0 - \nabla^2 H = -\epsilon_0\frac{\partial^2 B}{\partial t^2} \quad \because \nabla\cdot B = 0 \text{ or } \nabla\cdot H = 0 \ \&\ \nabla\times E = -\frac{\partial B}{\partial t}$$

$$-\nabla^2 H = -\epsilon_0\mu_0\frac{\partial^2 H}{\partial t^2} \quad \because B=\mu_0 H$$

$$\boxed{\nabla^2 H = \epsilon_0\mu_0\frac{\partial^2 H}{\partial t^2}}$$

EM wave in terms of D :–

$$\nabla^2 E = \mu_0\epsilon_0\frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2\left(\frac{D}{\epsilon_0}\right) = \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\left[\frac{D}{\epsilon_0}\right] \quad \because D=\epsilon_0 E$$

$$\boxed{\nabla^2 D = \mu_0\epsilon_0\frac{\partial^2 D}{\partial t^2}}$$

Standard form of wave equation

$$\nabla^2\begin{bmatrix}E\\H\\B\\D\end{bmatrix} = \mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\begin{bmatrix}E\\H\\B\\D\end{bmatrix}$$

$$\boxed{\nabla^2 A = \frac{1}{v^2}\frac{\partial^2 A}{\partial t^2}}$$ Where $A = \begin{bmatrix}E\\H\\B\\D\end{bmatrix}$ & $v^2 = \frac{1}{\mu_0\epsilon_0}$

$v = \frac{1}{\sqrt{\mu_0\epsilon_0}}$, $\mu_0 = 4\pi\times10^{-7}$ H/m
$\epsilon_0 = 8.85\times10^{-12}$ F/m

$$v = \frac{1}{\sqrt{4\pi\times10^{-7}\times8.85\times10^{-12}}} = 2.99792458 \text{ m/s} \simeq 3\times10^8 \text{ m/s}$$

Hence $\Rightarrow \boxed{v = c}$

It is proved here that EM waves are travelled with the speed of light.

Q:- Show that EM waves are travelled with the speed of light.

Q:- Show that EM waves are transverse in nature.

Soln:- Transverse Nature of EM waves:-

Let the electric field $E = E_0 e^{i(k.r-\omega t)}$ & magnetic field $H = H_0 e^{i(k.r-\omega t)}$

& $k.r = (\hat{i}k_x + \hat{j}k_y + \hat{k}k_z).(\hat{i}x + \hat{j}y + \hat{k}z) = (xk_x + yk_y + zk_z)$

Now find $\nabla.E = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).E_0 e^{i(k.r-\omega t)}$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).(E_{0x}\hat{i} + E_{0y}\hat{j} + E_{0z}\hat{k})e^{i(k_x x + k_y y + k_z z - \omega t)}$$

$$= \frac{\partial}{\partial x}\left[E_{0x}e^{i(xk_x + yk_y + zk_z - \omega t)}\right] + \frac{\partial}{\partial y}\left[E_{0y}e^{i(xk_x + yk_y + zk_z - \omega t)}\right]$$
$$+ \frac{\partial}{\partial z}\left[E_{0z}e^{i(xk_x + yk_y + zk_z - \omega t)}\right] = (iE_{0x}k_x + iE_{0y}k_y + iE_{0z}k_z)e^{i(k_x x + k_y y + k_z z - \omega t)}$$

$\nabla.E = i(k.E_0)e^{i(k.r-\omega t)} = i(k.E) \Rightarrow \nabla.E = i(k.E)$

$\nabla.E = 0 = i(k.E) \Rightarrow \boxed{k.E = 0}$

Similarly $k.H = 0$, Hence k is $\perp^r$ to E & H.
"EM waves are transverse in nature.

Relationship between electric & magnetic field of an EM wave :-

(i) Electric & magnetic fields are in phase. It means maxima of electric field concide the maxima of magnetic field. in time & space.

(ii) Electric & magnetic fields are $\perp^r$ to each other & EM wave propagats in $(E \times B)$ direction.

(iii) $E_0 = cB_0$

(iv) A change in magnetic field generates electric field & vice-versa.

Energy of an electromagnetic :-

We know that electric and magnetic fields are carried the equal energy in an electromagnetic wave.

So, $U = U_E + U_B$ = Energy due to Electric field + Energy due to magnetic field

$$\because U = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2$$

$$U = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\cancel{\mu_0}} \times \epsilon_0 \cancel{\mu_0} E^2 \quad \because B = \frac{E}{c} = E \times \sqrt{\mu_0 \epsilon_0}$$

$$\boxed{U = \epsilon_0 E^2} \text{ --- (1)}$$

Let $E = E_0 \cos(k \cdot x - \omega t)$ in x direction

Then $U = \epsilon_0 E_0^2 \cos^2(kx - \omega t)$ --- (2)

According to Poynting Theorem,

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})\hat{i} = \frac{1}{\mu_0} E_0 \cos(kx - \omega t) \times \frac{E_0}{c} \cos(k \cdot x - \omega t)\hat{i}$$

$$\vec{S} = \frac{1}{\mu_0 c} E_0^2 \cos^2(kx - \omega t) \times \frac{\epsilon_0}{\epsilon_0}\hat{i} = \frac{U}{\mu_0 \epsilon_0 c} = cU\hat{i} \quad \because \mu_0 \epsilon_0 = \frac{1}{c^2}$$

$$\boxed{\vec{S} = cU\hat{i}} \Rightarrow U = \frac{\vec{S}}{c} \text{ --- (3)}$$

Momentum of an electromagnetic wave :-

$$E^2 = U^2 = p^2c^2 + (m_0c^2)^2 \quad m_0 = 0$$

$$U^2 = p^2c^2 + 0 \Rightarrow U = pc \Rightarrow \boxed{p = \frac{U}{c} = \frac{S}{c^2}} \text{ --- (4)}$$

Radiated Pressure of an electromagnetic wave :-

Radiated Pressure (P_{rad}) $= \frac{F}{A} = \frac{1}{A}\frac{dp}{dt}$ from Newton's 2nd law

$$P_{rad} = \frac{1}{Ac}\frac{dU}{dt} \quad \text{from } (4)$$

We know that, energy radiated per unit area per unit time is called Poynting vector (S) $= \frac{1}{A}\cdot\frac{dU}{dt}$

$$P_{rad} = \frac{S}{c}$$

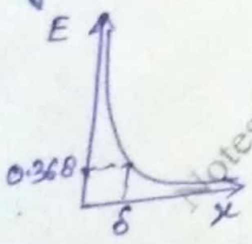
Now for perfectly reflecting surface, Resultant Pressure or

Total Radiated Pressure $= \frac{2S}{c}$

$$\boxed{P_{rad} = \frac{2S}{c}}$$

Because Total momentum
= incoming + reflecting momentum
$= \frac{2U}{c}$ Hence Prad will be twice.

Skin depth :- It is a measure of how deep electromagnetic radiation can penetrate into the material. Skin depth is also known as depth of penetration. It can also defined as the depth at which the strength of associated electric field reduces to 1/e times of its initial value. It is denoted by δ.



$$\delta = \frac{1}{\omega\left[\left(\frac{\mu\epsilon}{2}\right)\left(\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon^2}}\right) - 1\right]^{1/2}}$$

ω is the angular frequency.

* Skin depth is depend upon frequency. If $\frac{\sigma}{\omega\epsilon} \ll 1$ medium is dielectric & $\frac{\sigma}{\omega\epsilon} \gg 1$ medium is conductor.
